Morphological Image Processing

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Reference: Rafael C. Gonzalez, Richard E. Woods, "*Digital Image Processing,*" Second Edition, Prentice Hall, p.519-560&617-621

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1. Introduction

Morphology commonly denotes a branch of biology that deals with the form and structure of animals and plants.

Here, the same word *morphology* is used as a tool for extracting image components that are useful in the representation and description of region shape. It is also used for pre- or post processing, such as filtering.

The language of mathematical morphology use set theory to represent objects in an image.

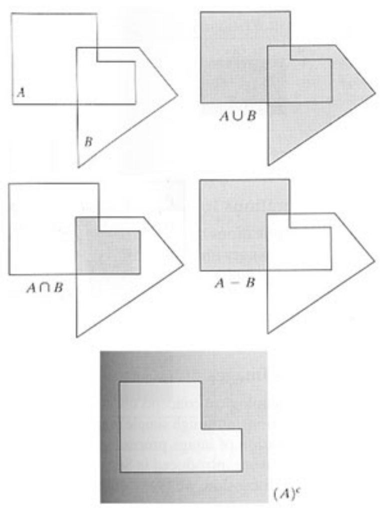
2. Preliminaries

- **Basic Concepts from Set Theory**

For binary image, let A be a set in Z^2

a ∈ A; a = (a1, a2) is an element of A.

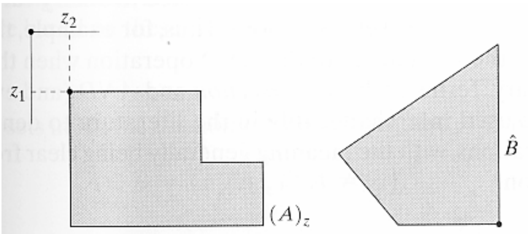
Set Operations:



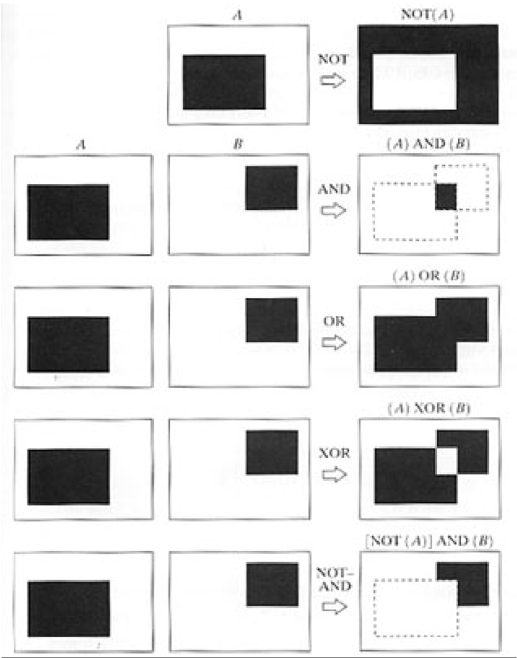
Addition Operation:

Reflection: $\hat{B} = \{w \mid w = -b, \quad \text{for} \quad b \in B\}.$

Translation: $(A)_z = \{c \mid c = a + z, \quad \text{for} \quad a \in A\}.$



- **<u>Logic Operations</u>**



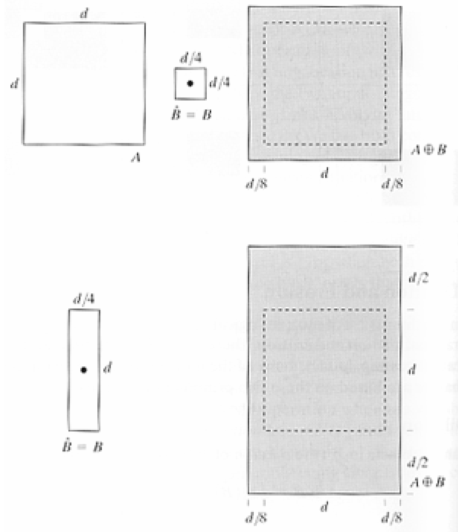
3. Morphological Operations

- **<u>Dilation and Erosion</u>**

Dilation: $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}.$

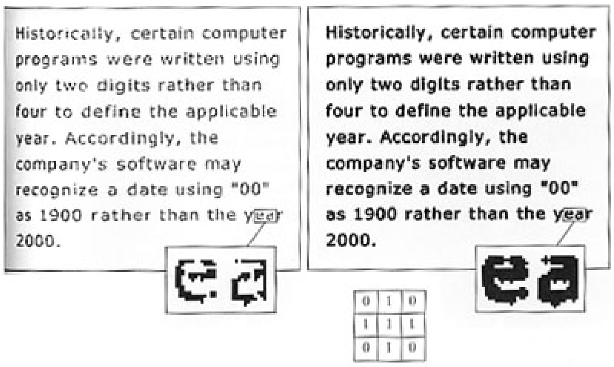
Set B is commonly referred to as the *structuring element*, and also viewed as a convolution mask.

Although dilation is based on set operations where convolution is based on arithmetic operations, the basic idea is analogous. B is flipping about its origin and slides over set (image) A.

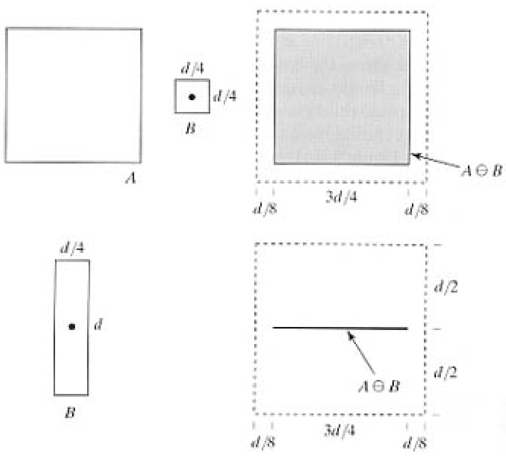


Dilation: Joining broken segments

One immediate advantage of the morphological approach over lowpass filtering is that the morphological method resulted directly in a binary image, while lowpass filtering started with producing gray-scale image.

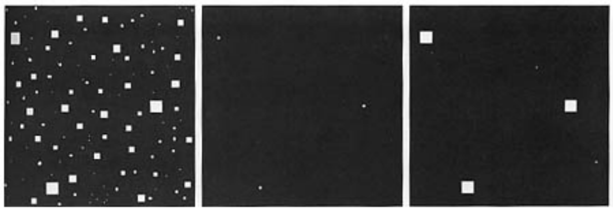


Erosion: $A \ominus B = \{z \mid (B)_z \subseteq A\}.$



Erosion & Dilation: eliminating irrelevant detail

Suppose we want to eliminate all the squares except largest one. We can do this by eroding the image with a structuring element of a size somewhat smaller than the objects we wish to keep. After that, we can restore it by dilating them with the same structuring element we used for erosion.

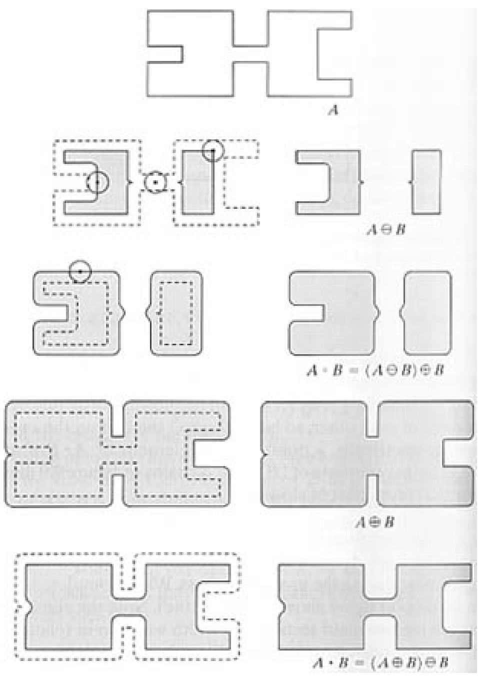


- **<u>Opening and Closing</u>**

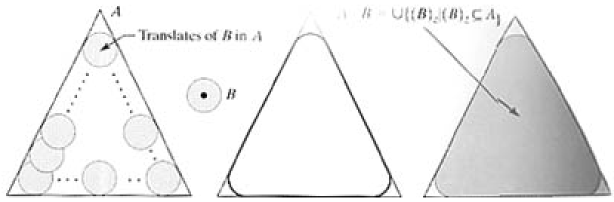
Opening generally smoothes the contour object, breaks narrow isthmuses, and eliminates thin protrusions. *Closing* also tends to smooth sections of contours but, ass opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

Opening: $A \circ B = (A \ominus B) \oplus B,$

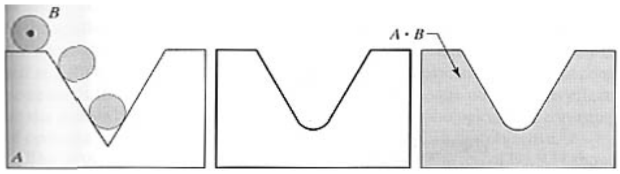
Closing: $A \bullet B = (A \oplus B) \ominus B,$



Opening: roll B around the *inside* of A.



Closing: roll B around the *outside* of A.



Opening & Closing: Noise Filter

The light elements are completely eliminated in first erosion stage, but unfortunately image is smaller so we have to restore it with dilation (erosion then dilation →opening of A by B).

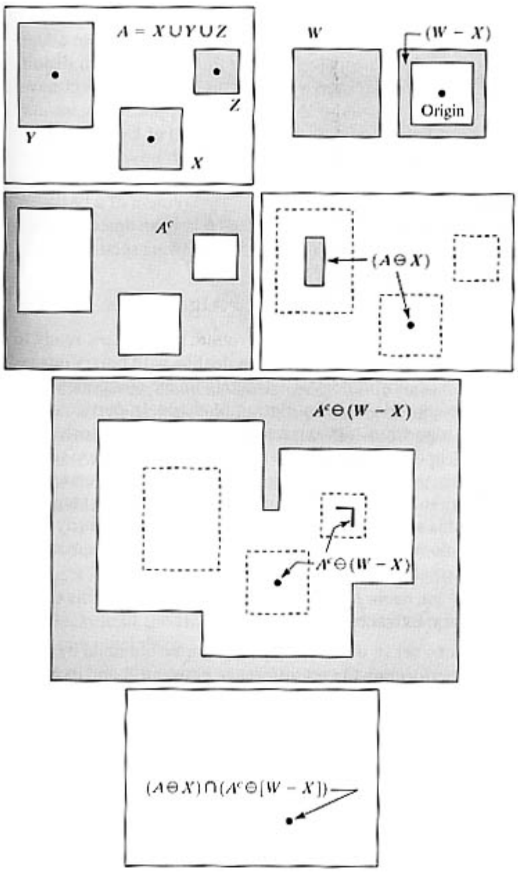
However, new gaps were created. To counter this effect we have to perform closing on an image again.

- **The Hit-or-Miss Transformation**

The Hit-or-Miss transform is a basic tool for shape detection. The objective is to find the location of one of the shapes in image.

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)].$$

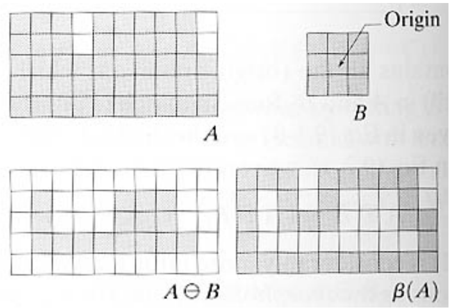
The small window, W, is assumed that have at least one-pixel-thick than an object. Anyway, in some applications, we may be interested in detecting certain patterns, in which case a background is not required.

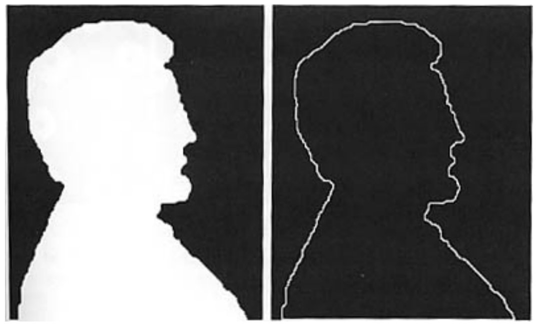


4. Basic Morphological Algorithms

- **Boundary Extraction**

$$\beta(A) = A - (A \ominus B)$$

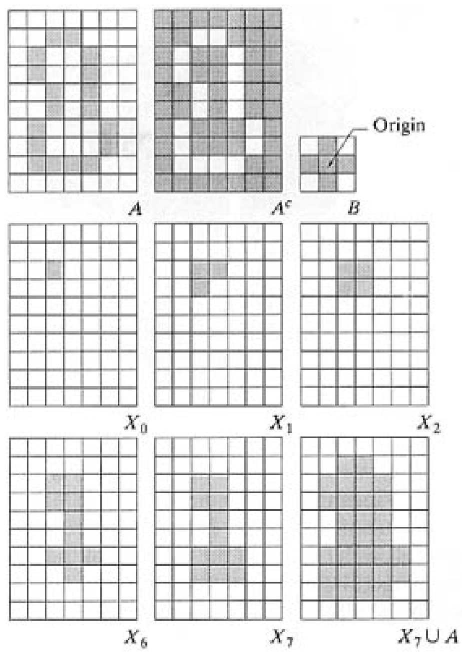




- **Region Filling**

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \ldots$$

Beginning with a point p inside the boundary, the objective is to fill the entire region with 1's, by iteratively processing dilation.

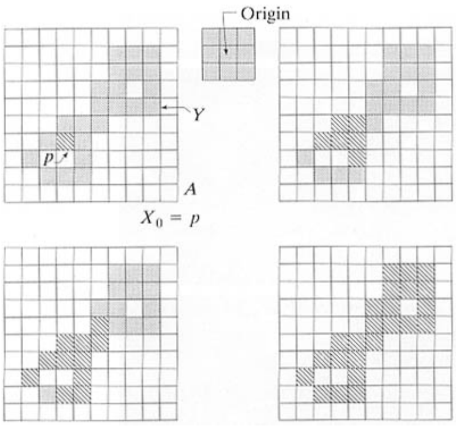


Adding the intelligence to detect a black inner point of sphere, we can use region filling to fill up the sphere to be completely white.

- **<u>Extraction of Connected Components</u>**

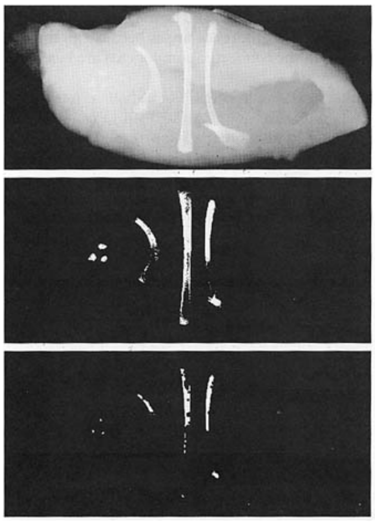
$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

The equation is similar to region filling. The only difference is the use of A instead of its complement.



Using connected components to detect foreign objects in packaged food.

After extracting the bones from the background by using a single threshold, to make sure that only objects of significant size remain by eroding the thresholded image. The next step is to analyze the size of the objects remain



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

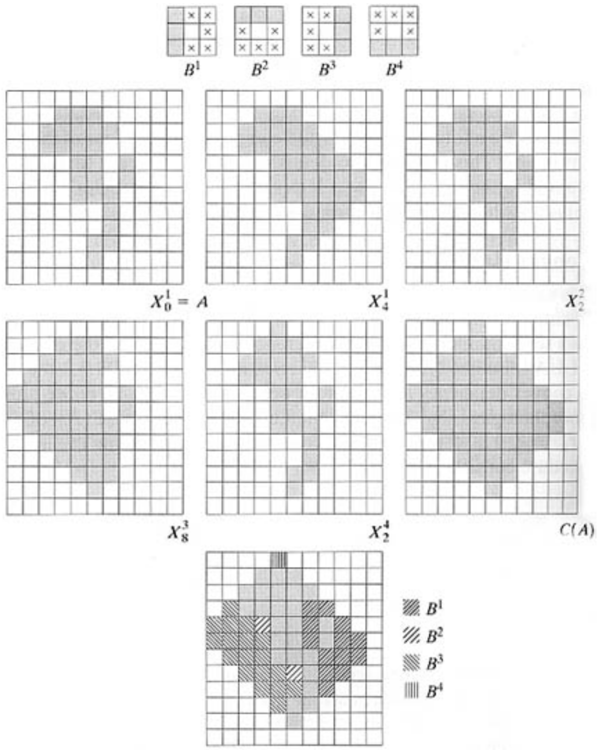
- **<u>Convex Hull</u>**

A is said to be *convex* if the straight line segment joining any two points in A lies entirely within A.

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \ldots$$

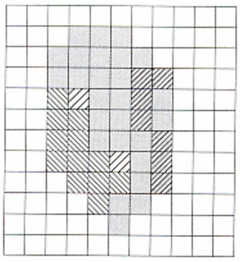
with $X_0^i = A$, and let $D^i = X_{\text{conv}}^i$ ("conv"→convergence)

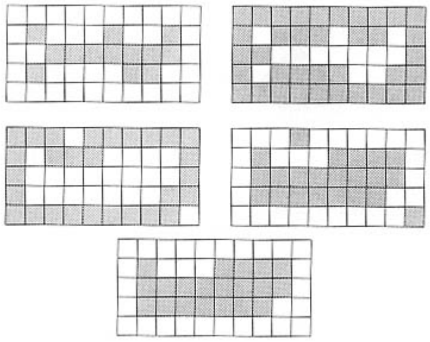
Then the convex hull of A is $C(A) = \bigcup_{i=1}^{4} D^i.$



In other words, the procedure consists of iteratively applying the hit-or-miss transform to A with B; when no further changes occur, we perform the union with A and call the result D.

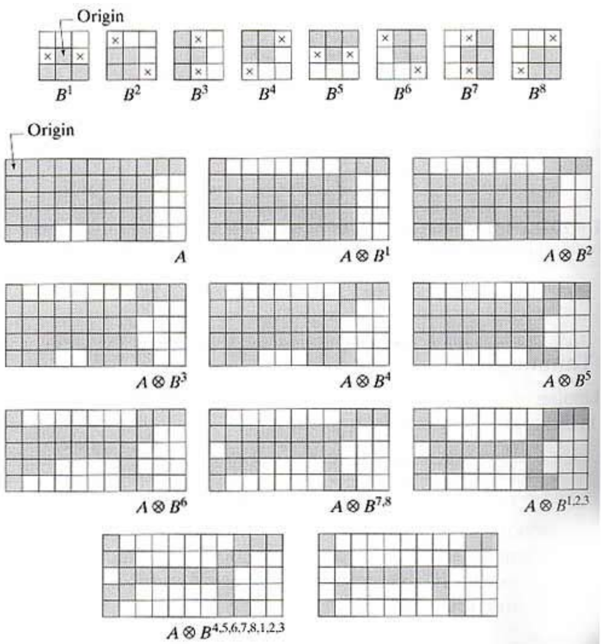
The limiting growth of convex can also be applied for better result.





- **Thinning**

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$



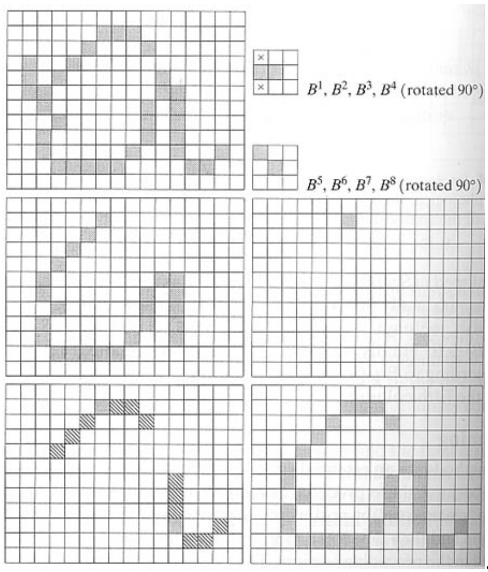
- **Thickening**

$$A \odot B = A \cup (A \circledast B)$$

The structuring elements have the same form as in *thinning* but with all 1's and 0's interchanged. However, a separate algorithm for thickening is seldom used in practice. The usual procedure is to thin the background instead.

- **Pruning**

Pruning methods are an essential complement to the procedures that tend to leave parasitic components that need to be "cleaned up" by post processing. For example, the automated recognition of hand printed characters.



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$X_1 = A \otimes \{B\}$ Thinning 3 times.

$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ End-point detectors.

$X_3 = (X_2 \oplus H) \cap A$ Grow line.

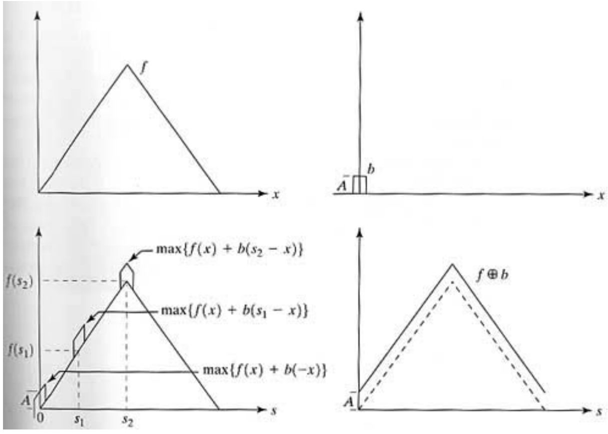
$X_4 = X_1 \cup X_3$ Restore the character.

5. Extensions to Gray-Scale Images

Throughout the discussions, we deal with digital image functions of the form *f(x, y)* and *b(x, y)*, where *f(x, y)* is the in put image and *b(x, y)* is a structuring element.

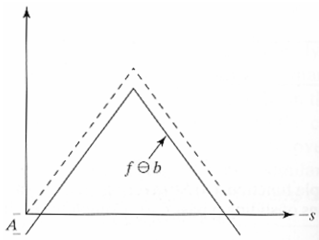
- **Dilation**

$$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$$

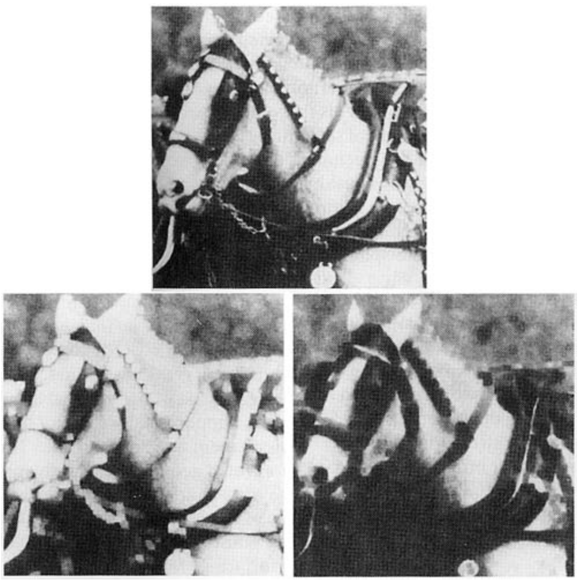


- **Erosion**

$$(f \ominus b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$



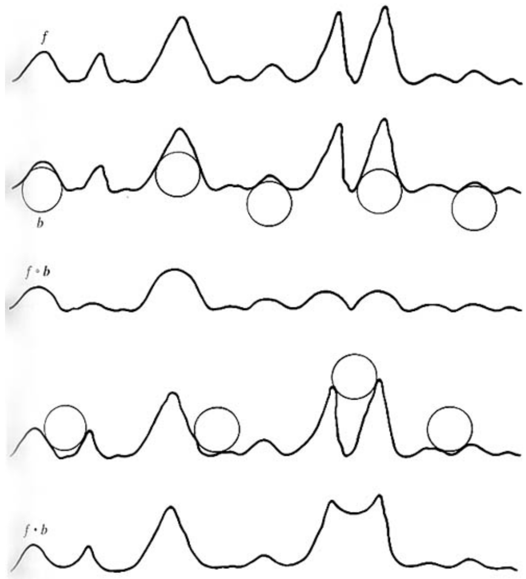
Dilation is expected to produce an image that is brighter that the original and in which small, dark details have been reduced or eliminated. In the other hand, erosion produces darker image, and the sizes of small, bright features were reduced.

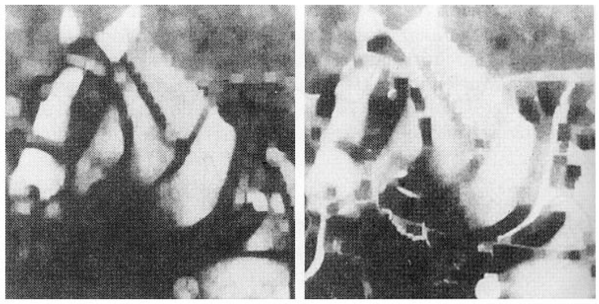


- **Opening, and Closing**

Opening: $f \circ b = (f \ominus b) \oplus b.$

Close: $f \bullet b = (f \oplus b) \ominus b.$



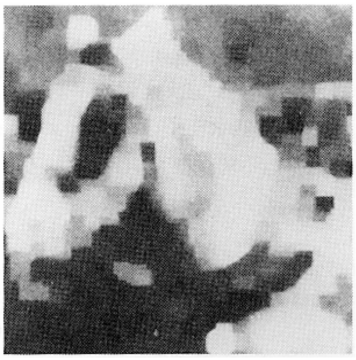


Note that opening decreases sizes of the small bright detail, with no appreciable effect on the darker gray levels, while the closing decreases sizes of the small dark details, with relatively little effect on bright features.

6. Some Applications of Gray-Scale Morphology

- **Morphological smoothing**

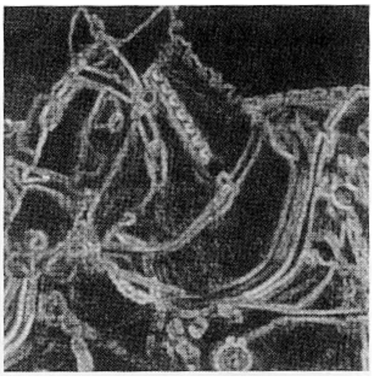
One way to achieve smoothing is to perform a morphological opening followed by a closing.



- **Morphological gradient**

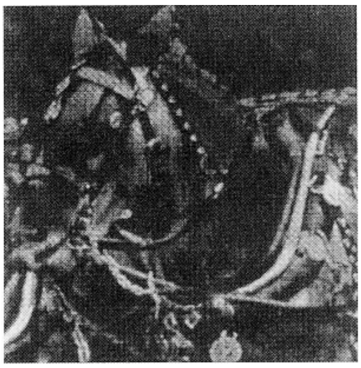
The morphological gradient highlights sharp gray-level transitions in the input image.

$$g = (f \oplus b) - (f \ominus b).$$



- **Top-hat transformation**

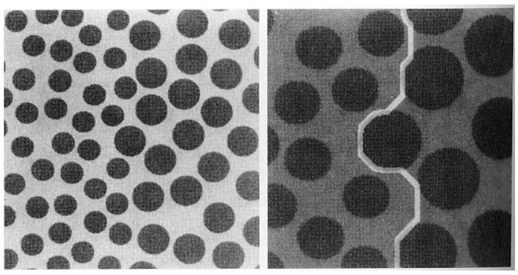
$$h = f - (f \circ b)$$



Note the enhancement of detail in the background region below the lower part of the horse's head.

- **Textural segmentation**

A simple gray-scale image composed of two texture region. The large blobs on right and small on left.

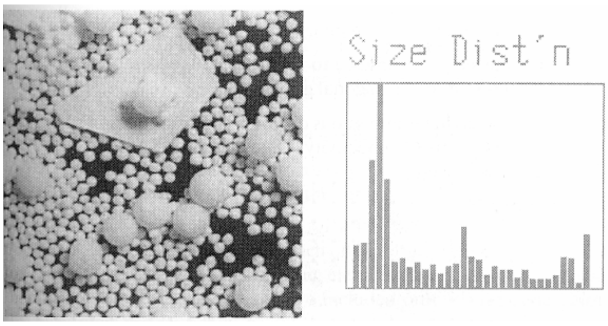


1. Closing with the small blobs, leaving left area with light background.
2. Opening with the large blobs, leaving a dark region on right.

⇒ The process has produced a light region on the left and a dark region on the right.

- **Granulometry**

Granulometry is a field that deals principally with determining the size distribution of particles in an image



As the particles are lighter than the background, we use opening with increasing size of structuring elements, and compute the difference between the original image and its opening. The histogram of that difference indicates the presence of three predominant particle sizes in the input image.

7. Summary

The morphological concepts constitute a powerful set of tools for extracting features of interest in an image. A significant advantage in terms of implementation is the fact that dilation and erosion are primitive operations.