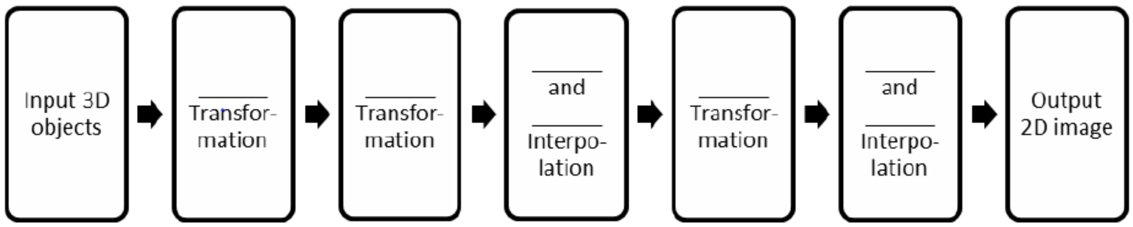
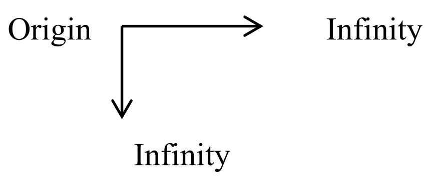
CS 112: Introduction to Computer Graphics (Winter 2020)
Written Assignment 1

1) Fill the blanks to complete the graphics rendering pipeline. [7]



2) Many image files have linear structure. If the size of the image is 1000x1000 (WxH), then the element at position 3500 in the file would refer to the element in the fourth row and 500^{th} column of the image. If the size of the image is infinite in both width and height, but has an origin (refer to the figure), then how would your organize your file into a linear structure? In other words, come up with a scheme of organizing the data in the file such that given the position of the element in the file, you can locate the corresponding position in the image. [4]



3) Consider a parallelepiped. A parallelepiped is a genus zero object with eight degree-three vertices and six parallelogram faces. (It is a sheared cuboid with no constraints on angle between the edges at a vertex.) Consider a 4x4 rigid model transformation matrix M. Naïve transformation of one vertex by M takes 16 multiplications and 12 additions/subtractions (4x4 matrix multiplied with a 4x1 vector). Hence for eight vertices, it would take 16x8 multiplications and 12x8 additions/subtractions. By utilizing characteristics of a rigid transformation, 16x___ multiplications and 12x___ additions/subtractions are enough to locate the transformed parallelepiped. [2+2=4]

a. 1
b. 2
c. 3
d. 8

4) One reason that homogeneous coordinates are attractive is that the 3D points at infinity in Cartesian coordinates can be explicitly represented by homogeneous coordinates. How can this be done? [3]

5) Consider a 2D square on the XY plane with side 2 units, the center at the origin and four sides parallel or perpendicular to the coordinate axes. Draw the picture of the transformed square after performing the following sequence of OpenGL commands. (Remember OpenGL post-multiplies the matrices in the order it is received, and finally the point is also post-multiplied.) (1.414 is the approximation of sqrt(2).)

glRotatef(45,0,0,1);
glTranslatef(1.414,0,0);
glRotatef(45,0,0,1);

Reduce the number of OpenGL function calls and thus give the new sequence of OpenGL function calls to effect the same transformation. [3+4=7]

6) Consider the same square as in Question 1 at the initial position. Draw the picture of the transformed square after performing the following sequence of OpenGL operations. [3+3+4=10]

Case 1:

glScalef(3,2,1);
glTranslatef(2,2,0);

Case 2:
Draw the picture of the transformed square if the above operations were swapped.

If we swap the function calls as in Case-2, but still want the output of Case-1, what parameters should we pass to the two commands? Fill in the blanks with correct parameters:

glTranslatef(____ , ____ , 0);
glScalef(___ , ___ , 1);

a. 1
b. 2
c. 3
d. 4
e. 6
f. 9

7) The inverse R^{-1} of a rotation matrix R is its transpose R^T. For R, the inter-relationship between different row vectors is ___ , and the inter-relationship between different column vectors is ___ . [2+2=4]

a. Parallel
b. Linear scaling
c. Orthogonal
d. Identical

8) In 3D, show $R_z(\theta1).\ R_z(\theta2) = R_z(\theta2).\ R_z(\theta1)$. What does this tell about the properties of rotation around coordinate axes? Show that $R_z(\theta1 + \theta2) = R_z(\theta1).\ R_z(\theta2)$. Using this property show that rotation about any arbitrary axis denoted by R_a also follows the property, $R_a(\theta 1).R_a(\theta 2) = R_a(\theta 2).R_a(\theta 1) = R_a(\theta 1 + \theta 2)$. [10]

9) Given notation a_{ij} means the entry of a matrix at *i*th row and *j*th column, the scaling matrix for scaling an object by a scale factor 3 along an arbitrary direction given by vector *u = (1, 2, 1)* rooted at *(5, 5, 5)* will be a_{00} = ____ , a_{01} = ____ , a_{02} = ____ , a_{03} = ____ , a_{10} = ____ , a_{11} = ____ , a_{12} = ____ , a_{13} = ____ , a_{20} = ____ , a_{21} = ____ , a_{22} = ____ , a_{23} = ____ , a_{30} = ____ , a_{31} = ____ , a_{32} = ____ , a_{33} = ____ .[16]

a. 0
b. 1
c. 2
d. 3
e. 5
f. -5
g. 6
h. 7
i. 10
j. -10
k. 25
l. -25

10) A viewer is defined by the following. (a) Eye position: *(0, 0, 0),* (b) View Up Vector: *(0, 2, 0),* (c) Equation of the image plane*: x+y+z = 6*. Find the matrix that will be generated by the function call **gluLookAt.** Let the left, right, top and bottom planes be at -2, +2, 4, and 8 respectively. Let the far plane be at 10. Find the perspective projection matrix given by the function call **glFrustum**. Find what would be projected coordinates of a point *P = (10, 4, 6)* for this viewer. [10+10+2=22]

11) The model transformation for our scene is a rotation R about the Y axis in the counter clockwise direction by 90 degrees, followed by a translation T in the positive X direction by 20 units. What is the resulting transformation? [3+3=6]

12) Choose the correct answer: If V is a vertex in our scene mentioned in Question 8, then after model transformation, the transformed vertex is computed as (a) RxTxV (b) TxRxV.[4]

13) The view transformation for our scene is the identity matrix. What is the position and orientation of the OpenGL camera? [5]

14) Consider three vertices A, B and C. Choose the normal vector of face ABC? [2]

a. |AB| * |AC|
b. (B - A) × (C - A)
c. (B - A) · (C - A)
d. ((B - A) + (C - B) + (A - C)) / 3

15) When is the vertex illumination computed in the OpenGL pipeline? ____ (a) Between the model-view and the projection transformations, or (b) After the projection transformation, before clipping. [2]

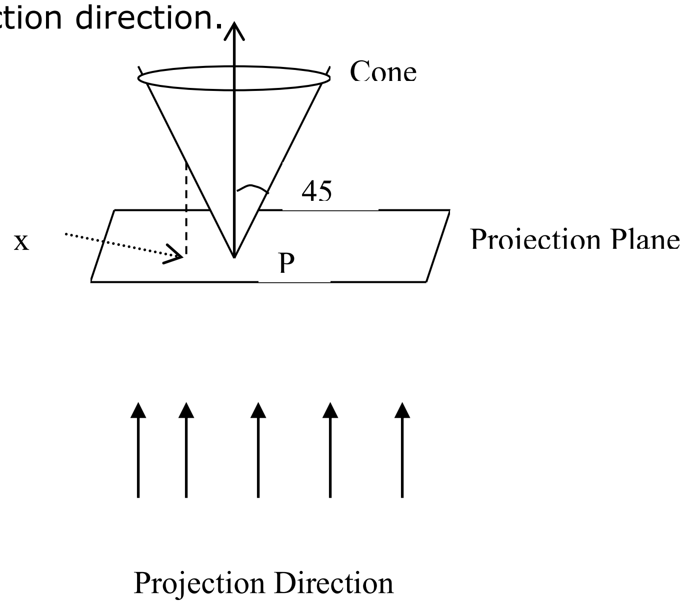
16) OpenGL can be instructed to cull (avoid rendering) triangles that are facing away from the viewer. Given viewing direction V and a triangle ABC both in world coordinates, assuming orthographic projection, how can you tell whether ABC is facing the viewer or not? What additional information do you need to compute the orientation of the triangle if we use perspective projection? [4+3=7]

17) You change the normal vectors of an OpenGL triangle, but don't change the position of the vertices. Which components of the color seen by the viewer might change? ____[2]

a. Ambient
b. Diffuse
c. Specular

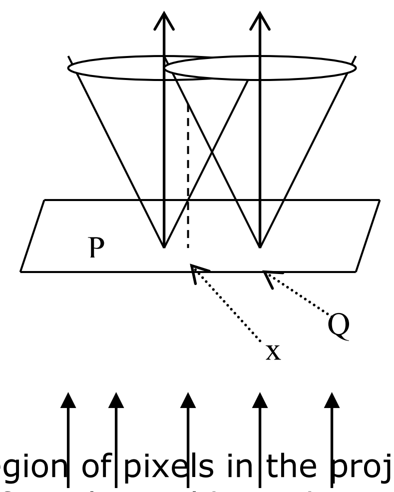
18) Definition: Silhouette edges are the edges in the manifold that have one back-facing polygon AND one front facing polygon incident on it. (1) How do you compute the silhouette edges of a manifold? (2) In OpenGL you can draw only back-facing polygons, or only front facing polygons. If you render the manifold(front facing polygons), then clear the frame-buffer but not the depth buffer, then again render only the back facing polygons. What do expect to see? (3) Assume that the "thickness" of a line is an attribute of a line. Thickness of three means that the line would be drawn three pixels "thick". In question (2), if the thickness of the line was one and now is increased to three only for the second rendering (rendering of back faces), what do you expect to see? [5+2+3=10]

19) Consider orthographic projection. The projection plane is perpendicular to the projection direction.



The surface of the cone makes an angle 45 degrees with the axis of the cone. The axis is parallel to the projection direction, and the apex (P) is on the projection plane. Assume that you are drawing the cone (and not the projection plane). What will be the depth value at any arbitrary point x on the plane? Express it in terms of distance(x, P), which is the distance between point x and P. [5]

20) Choose another point Q on the plane, and construct a similar cone as the one resting on P. The cone resting on P is colored red, and the one resting on Q is colored green. After projecting these two cones, the pixels on the projection plane would get the color of the point on the cone that is closer to the projection plane. For example, the vertical line at point x intersects the cone at P first and hence point x would get the red color. Use your answer to Question 12 and show that for each pixel which color should be assigned to it. [5]



21) Certain region of pixels in the projection plane would get red color and certain region of pixels would get the green color. Interpret the boundary of the regions with red cc … using your answer to Question 13. What will be the shape of the curve of this boundary (straight line, circle, ellipse, etc.)? [5]

22) Rasterize the line (P1, P2) where P1= (2,5), and P2=(8,15). Find the coordinates and the color of each pixel rasterized by this line segment, given the color of P1 is 0.8 and that of P2 is 0.1. Also show that the center of the pixel that is rasterized by this line is at most at a distance 0.5 from the actual line. [15+10+5 = 30]

23) Draw the results of clipping of a triangle ABC defined by A=(500,100), B=(800,460) and C=(400,500) against a window whose x_{min} = 300, x_{max}=700, y_{min}=200 and y_{max}=500, using Sutherland Hodgeman's method. Show the vertices remained in the window (including the ones newly created by clipping) for all the steps of the pipeline clearly. It does not matter if you do it clock-wisely or counter clock-wisely. [20]