Name:	PETER THE ANTEATER

Student ID:	C	S	1	1	2	-	Q	2

Pop Quiz (Week 3) [10 mins] – 14 pts

1) [3+3=6]Write the 4x4 matrix for following concatenated transformations.

a) $R_z(45°)$ $T(1,2,1)$

$$\begin{bmatrix} cos(45) & -sin(45) & 0 & 0 \\ sin(45) & cos(45) & 0 & 0 \\ 0 \quad 0 & & 1 & 0 \\ 0 \quad 0 & & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 3\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $S(1,2,1)$ $R_z(45°)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(45) & -sin(45) & 0 & 0 \\ sin(45) & cos(45) & 0 & 0 \\ 0 \quad 0 & & 1 & 0 \\ 0 \quad 0 & & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 0 \\ 2\frac{\sqrt{2}}{2} & 2\frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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2) [5] Consider a view setup where the eye is located at the origin, the normal to the image plane is given by the vector (0,1,1), and the view-up vector is given by (1,1,0). Find the view transformation.

The normal of the image plane will be the z-axis of our eye. Since we also know the up-vector, we can find the x-axis of our eye by taking the cross product of the normal and up-vector:

X_n = N x U = (0, 1, 1) x (1, 1, 0) = (-1, 1, -1)

Now, we can find the y-axis by taking the cross product of the normal (i.e. z-axis) and the x-axis:

Y_n = (-1, 1, -1) x (0, 1, 1) = (2, 1, -1)

Next, we must normalize the three vectors to become unit vectors. So, we have:

$$\|X_n\| = \sqrt{(-1)^2 + (1)^2 + (-1)^2} = \sqrt{3}$$

$$\|Y_n\| = \sqrt{(2)^2 + (1)^2 + (-1)^2} = \sqrt{6}$$

$$\|Z_n\| = \sqrt{(0)^2 + (1)^2 + (1)^2} = \sqrt{2}$$

Our normalized vectors are:

$$X'_n = \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

$$Y'_n = \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$$

$$Z'_n = \left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

We can now specify our view matrix:

$$\begin{bmatrix} \frac{-1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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3) [1+1+1=3] Mark all the correct answers for each of following questions.

a) A rigid body transformation:

- **i. Preserves lines as lines**
- **ii. Preserves lengths and angles**
- **iii. Preserves ratios of lengths and angles**
- **iv.** Can turn parallel lines to intersecting and vice versa

b) An affine transformation:

- **i. Preserves lines as lines**
- **ii.** Preserves lengths and angles
- **iii. Preserves ratios of lengths and angles**
- **iv.** Can turn parallel lines to intersecting and vice versa

c) A projective transformation:

- **i. Preserves lines as lines**
- **ii.** Preserves lengths and angles
- **iii.** Preserves ratios of lengths and angles
- **iv. Can turn parallel lines to intersecting and vice versa**